and the existence of local bases functions for bivariate spline spaces are discussed. Moreover some aspects of the approximation power and of interpolation by such spaces are given. Chapter 3 deals with nonlinear sets of splines (rational and regular splines) and their applications in solving ordinary differential equations. Chapter 4 deals with the numerical treatment of integral equations by using splines. The spline Galerkin method and the spline collocation method for solving Fredholm and Volterra integral equations and integro-differential equations are described. Chapter 5 describes the numerical solution of ordinary boundary value problems as well as delay differential equations with the aid of splines. The connection of collocation methods and methods of Runge–Kutta type are studied. In Chapter 6, some results on Lagrange and Hermite interpolation by finite elements functions are given. Chapter 7 is devoted to the solution of boundary value problems for partial differential equations. Finite element methods for solving elliptic Dirichlet and Neumann problems and spline collocation methods for solving parabolic and hyperbolic problems are studied. Chapter 8 describes the use of spline curves and tensor product surfaces in computer aided geometric design, in particular the approach of non-uniform rational B-splines. In Chapter 9 a shape model, which combines deterministic splines and stochastic fractals, is discussed. Variational splines in one variable are introduced and how to discretize the spline energy expressions using finite elements is described. In Chapter 10 properties of box splines and multivariate truncated power functions, in particular recurrence relations and the behavior of box spline series, are presented. Chapter 11 gives a brief introduction to spline wavelets, in particular to the use of univariate cardinal B-splines to generate a multiresolution analysis.

The book is addressed to researchers and scientists interested in applications. It gives an informative introduction to splines and to several fields of applications in which splines play a fundamental role. For a more detailed study of the topics, the reader is referred to the extensive list of about 6000 papers in the references.

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Sergei Yu. Slavyanov and Wolfgang Lay: *Special Functions, A Unified Theory Based on Singularities*, Oxford Mathematical Monographs, Oxford University Press, 2000, xvi + 293 pp.

The central issue of this monograph is Heun's differential equation. This equation generalizes the hypergeometric equation by the addition of one more finite regular singularity. The equation itself never really penetrated into the domain of physical sciences, except for the case of Lamé's equation, where the singularities are specialized to be elementary. The additional singularity, however, creates the possibility of several (and new) types of confluences, and exactly the confluent versions of the equation show up in many different fields (e.g., the spheroidal wave equation, Mathieu equation, several Schrödinger cases), often giving rise to new types of eigenvalue problems. The class of Heun equations is presented here in a unifying, singularity-based, framework. On one side, the class is extending the hypergeometric class in the above-mentioned sense, while in another direction, the equations are shown to be in one-to-one correspondence with the nonlinear equations of the Painlevé class.

Chapter 1 introduces the relevant tools from the general theory of linear second-order ODEs with polynomial coefficients: *s*-rank classification and principles of confluence, Frobenius versus Thomé expansions, the generalized Riemann scheme, *s*-homotopic, Möbius, and Jaffé transformations, the central two-point connection problem, and the Birkhoff set of asymptotic solutions to the Poincaré–Perron difference equation for the expansion coefficients. In Chapter 2 these tools are applied to reconsidering the hypergeometric class in the unifying framework. The chapter includes an original treatment of the difference equations satisfied by hypergeometric and confluent hypergeometric functions and a section about the related classical orthogonal polynomials. Following the same lines, the Heun class of equations is extensively dealt with in Chapter 3. Important sections here are devoted to the case of nearby singularities, to the phenomenon of avoided crossings in eigenvalue problems for the triconfluent Heun equation, and to the two-parametric eigenvalue problem arising with the doubly confluent Heun equation. Chapter 4 presents a selection of (eigenvalue) problems in physics, astrophysics, and celestial mechanics where confluent versions of Heun's equation show up. Chapter 5 is devoted to the one-to-one correspondence between Heun-class equations (interpreted as Hamiltonians) and equations from the Painlevé class (interpreted as equations of motion).

The book is concisely written, omitting long proofs and reducing critical ones to the essentials. With its numerous tables, lists, and schemes it can be considered as a useful reference work for researchers in applied mathematics and physics. The software package SFTools, announced in Appendix D, might be of special interest to them.

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Yves Nievergelt, *Wavelets Made Easy*, Birkhäuser, Boston, 1999 (2nd printing with corrections 2001), xi + 297 pp.

During the past few years several books on wavelets have appeared. The book under review deserves special attention because it really lives up to what its title promises: it makes wavelets easy. The author claims that the material presented addresses the audience of engineers, financiers, scientists, and students looking for explanations of wavelets at the undergraduate level. The more advanced researcher, and in particular approximation theorists, will need supplementary material, but will nevertheless like the lucid presentation.

The book consists of three parts, A, B, and C (and a mysterious part D called *Directories* which consists of an empty page). Part A (Chapters 1–3) deals with *Algorithms for Wavelet Transforms*. The author starts with a chapter devoted to Haar wavelets, which are worked out in detail. Very little calculus or linear algebra is needed to explain these Haar wavelets, but they allow a clear explanation of the general nature of wavelets and the fast wavelet transform, illustrated by examples from creek water temperature analysis and financial stock index event detection. In Chapter 2 the simple Haar wavelet is extended to two-dimensional Haar wavelets, with the help of some linear algebra. Applications in noise reduction, data compression, and edge detection are explained, with some computational notes, and some extra examples on two and three dimensional diffusion analysis are given. Chapter 3 deals with Daubechies wavelets (in one or more dimensions), with emphasis on the calculation (the fast Daubechies wavelet transform). This chapter also shows the need for some theory for the clarification of Daubechies wavelets, and this will be done in parts B and C.

Part B (Chapters 4–6) is about *Basic Fourier Analysis* and deals with the classical theory of least squares approximation with trigonometric functions. In fact, wavelets hardly appear, but it is clear that the results and techniques of this part will be used in part C for the computation and the design of wavelets. Chapter 4 gives the basic ingredients of Fourier analysis, such as inner product spaces, Gram–Schmidt orthogonalization, and orthogonal projections, and it is shown how orthogonal projections are used in three-dimensional computer graphics, in least-squares regression, in the computation of functions, and (of course) for building wavelets. Chapter 5 describes the discrete Fourier transform and its computation using the fast Fourier transform technique of Cooley and Tukey. The theory of Fourier series for periodic functions is worked out in Chapter 6, with some notions of convergence and inversion of Fourier series (such as the Gibbs–Wilbraham phenomenon).

As mentioned above, part C (Chapters 7–9) is about the *Computation and Design of Wavelets* and shows how the Fourier analysis of part B is used in wavelet design. Chapter 7 presents the Fourier transform and its inversion on the line and in space. In Chapter 8 it is